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Momentum-Balance Aspects of Free-Settling Theory. II. Continuous, Steady-State Thickening

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Momentum-Balance Aspects of Free-Settling Theory. II. Continuous, Steady-State Thickening

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Abstract

The theory of steady-state, continuous thickening in the free-settling concentration range is considered, taking into account the momentum balance relation governing the process. It is concluded that there is no solids capacity limitation associated with the free-settling portion of the thickening zone.

INTRODUCTION

In a previous paper (*1*) an analysis of batch thickening of an initially free-settling slurry was given. This analysis was based on consideration of the forces acting on the solid phase instead of the usual assumption that the settling rate is a function of concentration. The main conclusion reached was that, starting with a uniform, free-settling suspension, the free-settling zone will remain at the initial concentration, with a discontinuity at the interface between free-settling and compression zones, and a concentration gradient will develop only in the compression zone. No concentration gradient develops in the free-settling zone because of the absence of retarding forces which are necessary if thickening is to occur.

The present discussion applies the same approach to the process of

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steady-state, continuous thickening. It is found, again, that the conclusions reached differ in a basic way from those obtained from previous analyses.

The following discussion is based on the same five assumptions as employed previously, namely:

(1) The container has constant cross-sectional area.
(2) The slurry properties are uniform and constant; that is, the flocs are of uniform size (no segregation of different-sized particles) and solid and liquid properties are constant (isothermal conditions).
(3) The flow is vertical, and horizontally uniform (negligible wall effect).
(4) The forces which can act on the solid particles are gravity (allowing for buoyancy), liquid drag due to motion relative to the liquid, and interaction forces exerted by adjacent solids.
(5) The slurry can be treated as a continuum; that is, a continuous liquid phase and a continuous solid phase which interact with each other.

Below a certain "critical" concentration there are no solid–solid interaction forces and the slurry is said to be in "free settling." Above the critical concentration the slurry is in "compression."

The horizontal uniformity assumption (No. 3) requires more comment here than in the batch-thickening case. In a continuous thickener the feed is separated into two product streams; the clear liquid overflow and the thickened sludge underflow. Thus the flow cannot be horizontally uniform over the whole depth of the thickener. In the "clarification" or "separation" zone where the overflow and underflow streams are separating from each other, the former moving upward and the latter downward, the flow cannot be horizontally uniform. Hence the horizontal uniformity assumption implies that the separation zone is not being considered, but only the portion of the thickener below this, where all flow (liquid and solid) is downward. We will refer to this as the "thickening zone" and note that, even when all flow is downward, the assumption of horizontal uniformity is still an approximation.

The main concern in analyzing continuous thickeners is with their solids handling capacity when treating a given slurry, which can be exexpressed as the solids throughput rate per unit cross-sectional area (solids flux) for specified underflow concentration. As indicated in the preceding paragraph, the present discussion will deal with the capacity of the thickening zone and will not consider the question of the separation zone capacity. The basic theory of thickening zone capacity is that of Coe and Clevenger (2), which will be reconsidered in terms of force action in the thickening process.

COE AND CLEVENGER THEORY

Coe and Clevenger assumed that the limit to the thickening zone solids flux lies in the free-settling concentration range rather than in the compression range. The basis of this assumption was not made clear but, as interpreted by Fitch (3), it was tacitly assumed that settling rates in compression could be increased as necessary by increasing the depth of the compression zone, so that no limit existed in that zone. In the free-settling concentration range they assumed that the solids settling velocity was determined solely by their concentration, from which it follows that the solids settling flux (ϕ_r), relative to the flux induced by the bulk movement of the slurry, is also a function of concentration.

The solids material balance for a steady-state continuous thickener leads simply to the result that solids flux (ϕ), relative to the thickener, is the same at all levels. ϕ is determined by the feed rate of solids to the thickener, assuming that no solids leave in the overflow. The total material balance shows that the total flux (ϕ_t) is also the same at all levels in the thickening zone, and this is determined by the sludge pumping rate. Thus the solids settling flux is related linearly to the concentration, through the thickening zone, since it is given by

$$\phi_r = \phi - f\phi_t \tag{1}$$

ϕ and ϕ_t being constants determined by the operating conditions.

Thus Coe and Clevenger argued that the material balances demand a certain settling flux at each concentration involved in the thickener (given by Eq. 1), and if overloading is to be avoided, the flux requirement at each free-settling concentration involved (i.e., between feed and critical concentrations) must not exceed the inherent settling flux of the slurry at that concentration. This led to their method for determining the minimum area requirements for the thickener for given solids throughput and underflow concentration ($f_u = \phi/\phi_t$).

A graphical representation of the Coe and Clevenger analysis was introduced by Yoshioka et al. (4). On a plot of ϕ_r versus f the relation between ϕ_r and f dictated by the material balances (Eq. 1) is a straight line (the "operating line") with intercept ϕ on the ϕ_r axis, f_u on the f axis, and slope $-\phi_t$. The relation between ϕ_r and f determined by the inherent settling velocity–concentration relation for the slurry (the "flux line") starts at the origin and, after passing through a maximum, decreases with an increase in concentration. Figure 1 shows a hypothetical flux line and two operating lines to illustrate the following discussion. The flux line is terminated at the critical concentration, since the Coe and Clevenger method is only applied to the free-settling range. The feed concentration is indicated by f_f.

Another way of stating the Coe and Clevenger criterion is that the operating line must lie below the flux line everywhere in the range from f_f to f_c, as in the case of operating Line 1. According to their theory, the thickener could not operate at steady state with operating Line 2 because, in the range where the operating line is above the flux line, the settling flux demanded by the operating line exceeds that which the slurry can transmit. Hence, if the feed and sludge pumps were operated at rates corresponding to operating Line 2, solids would back up in the thickener until they were eventually carried out with the thickener overflow.

Some aspects of the Coe and Clevenger theory were not clearly elab-

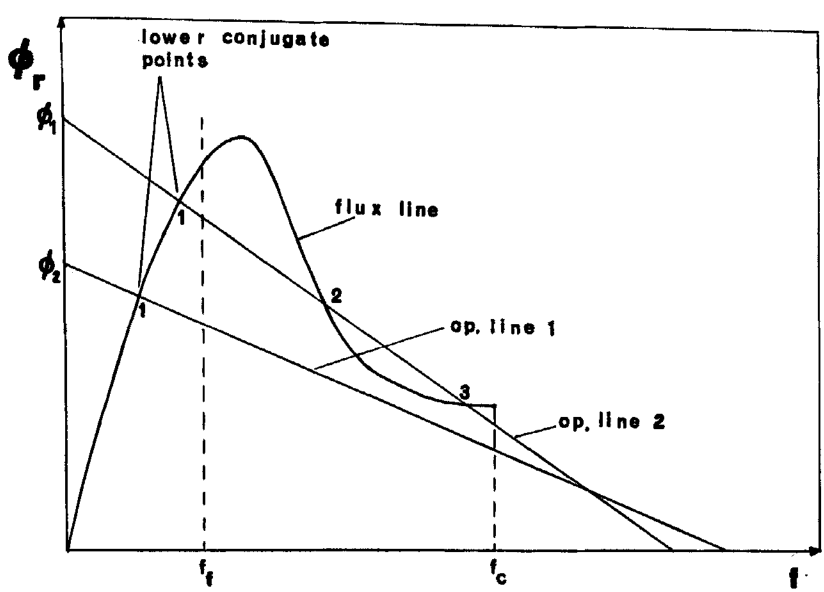

FIG. 1. Illustrative flux plot: free-settling range.

orated in the original paper and were not made clear until some years later. One question which arises is illustrated by operating Line 1 on Fig. 1. Everywhere in the range f_f to f_c operating Line 1 lies below the flux line, and so the settling capacity of the slurry is nowhere exceeded. However, not only is the operating flux everywhere not greater than the inherent flux, it is also less everywhere. What makes the solids settle less rapidly than their inherent rate?

Clearly, if $u = u(f)$, then there can be no way in which the solids will settle either faster or slower than u for given f, and the only free-settling concentrations which can exist in the thickener are those at which the operating and flux lines intersect (*5*). Thus, when operating according to Line 1, there will be no horizontally uniform free-settling zone.

Operating Line 2 has two intersections with the flux line in the range f_f to f_c (labeled 2 and 3), and it appears, therefore, that these two concentrations might appear in the thickener, presumably with a concentration discontinuity between them. Coe and Clevenger, however, concluded that steady operation according to Line 2 would not be possible, but it was Kynch (*6*) who gave a clearly stated reason for this. [As remarked by Dick and Ewing (*7*), the Kynch theory is the logical precursor of that of Coe and Clevenger.] Kynch's theory stated that the discontinuity demanded by operating Line 2 is unstable and would give rise to continuous movement of concentration zones upward, leading to thickener overflow.

INCLUSION OF THE MOMENTUM BALANCE IN THE ANALYSIS

The material and momentum balance equations for thickening, based on Assumptions 1 to 5, have been given previously (*1*). In the present case of steady-state thickening the equations simplify considerably. As noted above, the solids material balance reduces to ϕ = constant. The solids momentum balance reduces to

$$\rho v \frac{dv}{dx} = F_g + F_d - \frac{1}{f}\frac{d\tau}{dx} \tag{2}$$

Using ϕ = constant, this can be rewritten as

$$\rho \frac{v^2}{f}\frac{df}{dx} = -\left[F_g + F_d - \frac{1}{f}\frac{d\tau}{dx}\right] \tag{3}$$

Since $\rho v^2/f$ is always positive, Eq. (3) shows that for df/dx to be positive (i.e., for thickening to occur) the net force per unit volume acting on the solids must be negative; that is, the solids must be subject to a retarding force. This follows also from the fact that $\phi = fv$ is constant through the thickening zone. As f increases, v decreases, and this requires a retarding force (second law of motion).

In the free-settling concentration range, τ is zero, by definition, and Eq. (3) reduces to

$$\rho \frac{v^2}{f} \frac{df}{dx} = -(F_g + F_d) \tag{4}$$

The requirement for thickening is then that the drag force (which is negative) must be numerically greater than the gravitational force. As discussed previously (*1*), the assumption that settling velocity is a function of concentration in free settling is equivalent to assuming that the drag force is a function of concentration and relative velocity (Assumption 6), and that inertial effects may be neglected (Assumption 7). Thus, on the ϕ_r–f plot, the flux line can also be interpreted as the locus of points [each (f, ϕ_r) point defines an (f, u) point] for which the gravitational and drag forces are balanced. At all points above the flux line, drag exceeds gravity; at all points below the line, drag is less than gravity.

Hence, referring again to Fig. 1, for both operating lines the feed-concentration point on the operating line lies below the flux line. At such a point $F_g + F_d$ is positive and df/dx is negative (Eq. 4). That is, the only tendency would be for a decrease in concentration toward the "lower conjugate" concentration (Point 1 on each operating line). Once the lower conjugate concentration is reached (and this would be achieved through a very small depth, since acceleration or retardation is very rapid in free settling of small particles), $F_g + F_d$ is zero and no further concentration change will occur until some other retarding force comes into effect. Thus, if a horizontally-uniform free-settling zone is formed, its concentration will be the lower conjugate concentration. This will also apply if the feed concentration is lower than the lower conjugate concentration. The feed concentration would then lie on the operating line above the flux line, and so the concentration would increase toward the lower conjugate concentration. In free settling there is no driving force for thickening other than toward the lower conjugate concentration.

In the case of operating Line 2, the free-settling zone concentration might tend toward Point 3, instead of Point 1, if the feed concentration were high enough. Perhaps Point 1 would be approached if f_f is less than the Point 2 concentration, and Point 3 approached for greater concentrations. However, this could be influenced by behavior in the separation zone, which is beyond the scope of the present discussion. (There would be no tendency to approach Point 2 because df/dx is negative to the left of this point and positive to the right.)

Thus consideration of the fact that an increase in solids concentration is accompanied by a decrease in solids velocity, and that this requires a retarding force, leads to the conclusion that continuous thickening does not occur in the free-settling concentration range. This corresponds to the conclusion reached in the discussion of batch thickening that concentration gradients cannot develop in the free-settling zone.

For thickening to occur, compressive effects must come into effect, and this occurs when the free-settling solids reach the sediment and are retarded by impact, their concentration jumping to the critical concentration. The factors involved in this process are exactly the same as at the initial-concentration/sediment zone interface in batch thickening, discussed previously (*1*). It is of no consequence if the operating line intersects the flux line between the conjugate concentration and the critical (as in the case of Line 2), since in the impact retardation only inertial and compressive effects are involved, gravitational and drag forces (which the flux line relates) playing no significant part.

Thus the only conclusion that can be reached is that there is no flux limitation associated with the free-settling concentration range in the thickening process. This is the basic result obtained from reconsideration of the theory of continuous, steady-state free settling, taking force action into account, and it disagrees with the long-accepted Coe and Clevenger concept of a flux limitation associated with the inherent settling capacity of the slurry. The remainder of the discussion will deal with flux limitations in the compression zone (which are already treated in the literature), and will compare the overall conclusions with experimental results.

COMPRESSION ZONE FLUX LIMITATIONS

In the compression concentration range the flocs are in contact, and the application of a compressive stress is necessary to produce thickening. (In free-settling, no compressive stress is required for increase in concentration, and this allows the sudden jump to the critical concentration when the free-settling solids strike the top of the sediment.*) Because of

*Because of this statement, the writer believes that the concept of free settling is itself an approximation. Even before the flocs come into contact is seems that compressive stress (presumably very small) will be necessary to produce an increase in concentration.

this, a compression zone builds up in the thickener to such a depth that the compressive stresses necessary to produce the required underflow concentration are obtained. [The presence of a stress gradient in the compression zone (cf. Eq. 3) makes it possible for the solids to experience a net retarding force, as required for thickening to occur.] However, it is possible for the operating conditions to be such that the required compression zone depth is greater than the depth of the thickener, so that overloading occurs. This has been shown by Fitch (*5*) and later writers, and the considerations involved will be re-presented here so as to complete the discussion.

It seems reasonable to assume that the greater the underflow concentration, the greater the compressive stress must be at the bottom of the thickener, and that in general the stress must increase with depth through the compression zone. Rearrangement of Eq. (3) gives

$$\frac{1}{f}\frac{d\tau}{dx} = F_g + F_d + \frac{\rho v^2}{f}\frac{df}{dx} \tag{5}$$

The third term on the right-hand side of this equation is the inertia term, and it will be positive if thickening is taking place (df/dx positive). However, this term will normally be very small compared to the other right-hand side terms. Hence Eq. (5) shows essentially that for the compressive stress to increase with depth, $F_g + F_d$ must be positive; that is, the operating line must lie below the flux line in the compression concentration range. At any point on the operating line, $F_g + F_d$ is fixed and is that part of the unbuoyed weight of solids (per unit volume of solids) which is not supported by liquid drag. The nearer the operating line is to the flux line, at a given concentration, the less solids weight is unsupported by drag, and so the less rapidly the compressive stress increases with depth.

Figure 2 shows a hypothetical flux line for the compression concentration range and two operating lines to illustrate the following discussion. Operating Line 1 passes close to the flux line, and so the rate of increase of concentration with depth in the vicinity of this "squeeze region" is expected to be smaller than on each side of the region, since $d\tau/dx$ is smaller, because $F_g + F_d$ is smaller. The general shape of the expected depth versus concentration curve is also shown on Fig. 2. Experimental data of Comings (*8*) and later workers indicate that sludges are easier to compress in the vicinity of the critical concentration, but rapidly become more difficult to compress as the concentration increases. Thus the depth-concentration curve shown for Line 1 starts at the critical concentration with large df/dx, df/dx reduces as the squeeze region is passed,

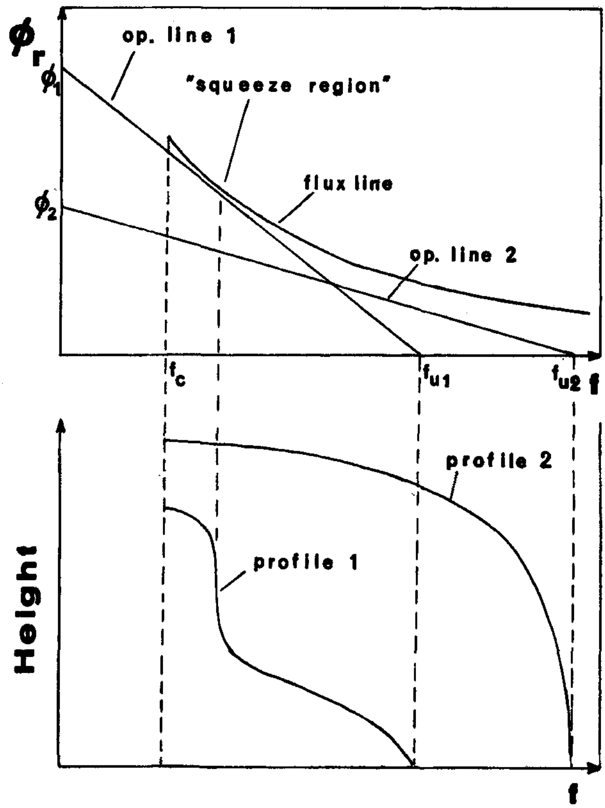

FIG. 2. Illustrative flux plot and concentration profiles: compression range.

then increases again, but df/dx decreases again at higher concentrations due to decreasing compressibility as concentration increases. As the operating line is moved closer to the flux line (say, by increasing the solids feed rate while maintaining the sludge withdrawal rate), the deeper the squeeze region will become; and in the limit it will be infinitely deep, so that overloading must occur at some feed rate. This corresponds to one of the two modes of thickener overloading, first reported by Comings (*8*), in which a nearly uniform zone of intermediate concentration appears in the thickener, resulting from "overfeeding."

Operating Line 2 on Fig. 2 does not pass close to the flux line, and so does not show a squeeze region in the depth-concentration curve, but the underflow concentration is higher and this requires a higher stress at the bottom of the thickener. Thus, even without the operating line passing close to the flux line, the required depth will increase if, say, the solids feed rate is maintained while the sludge withdrawal rate is decreased (corresponding to an increase in the steady-state underflow concentration). Thus a given thickener can be overloaded by sludge "underwithdrawal" without a uniform zone appearing, and this was the second mode of overloading observed by Comings.

DISCUSSION AND CONCLUSIONS

Taking into account force action during the thickening process leads to the conclusion that there is no solids flux limitation associated with the free-settling part of the thickening zone in a continuous thickener. The essence of the argument is that in free settling the retarding force necessary for thickening is not available, and thickening does not start until the solids are retarded by impact at the top of the sediment. Further thickening occurs as the solids pass through the sediment under an increasing compressive stress exerted by the solids above.

Consideration of the process in the compression zone shows that overloading will occur if the compression zone depth requirement is larger than available, and two modes of overloading (overfeeding and underwithdrawal) can be predicted, in qualitative agreement with experimental observations.

As in the previous discussion of batch thickening (*1*), available experimental data cannot show whether the present or previous analysis is correct. Thickener capacities predicted by the Coe and Clevenger method are not found to agree well with experimentally determined values (*9*, *10*), but this could be due to uncertainties in the experimental data on which the predictions are based. The uniform zone which appears on overloading by overfeeding is interpreted by the Coe and Clevenger theory as a free-settling concentration whose inherent settling flux limits the thickener capacity. The present analysis concludes that this concentration is a compression concentration whose rate of increase with depth is very low because nearly all the weight is supported by liquid drag. Hence the experimental problem is to determine whether the concentration zone in question is in the compression range or not, and this is a difficult task.

Thus, based on Assumptions 1 to 6 above, theoretical argument leads to the conclusion that capacity limitations in the thickening zone of a continuous thickener (distinguished from the separation zone) are associated with the compression zone rather than the free-settling zone.

SYMBOLS

The positive direction is downward for all vector quantities

f	solids concentration, volume fraction, dimensionless
f_c	critical solids concentration
f_f	feed solids concentration
f_u	underflow solids concentration
F_g	net gravitational force acting on the solids, per unit volume of solids, $N/m^3 = g(\rho - \rho_l)$
F_d	liquid-drag force acting on the solids, per unit volume of solids, N/m^3
g	acceleration due to gravity, m/sec^2
t	time, sec
u	velocity of the solids, relative to the slurry volume-average velocity, $m/sec = v - \phi_t$
v	velocity of the solids relative to the thickener, m/sec
w	velocity of the liquid relative to the thickener, m/sec
x	distance below stationary reference plane, m

Greek

ρ	solids density, kg/m^3
ρ_l	liquid density, kg/m^3
τ	solids compressive stress, based on total cross section, N/m^2
ϕ	volumetric flux of solids, relative to thickener, m/sec
ϕ_r	volumetric flux of solids, relative to flux induced by bulk flow, $m/sec = \phi - f\phi_t$
ϕ_t	total volumetric flux, m/sec = volume-average velocity of the slurry

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